 Failure Detection in Umbilicals via Electroactive Elements
A Mathematical Homogenization Approach

Leslie D. Pérez-Fernández\textsuperscript{1} and André T. Beck\textsuperscript{2}

leslie.fernandez@ufpel.edu.br, atbeck@sc.usp.br

\textsuperscript{1}Department of Mathematics and Statistics, Institute of Physics and Mathematics, Pelotas Federal University, Pelotas – RS, Brazil
\textsuperscript{2}Department of Structures Engineering, São Carlos School of Engineering, University of São Paulo, São Carlos – SP, Brazil

Abstract. By means of a simple example, it is illustrated the idea that global failure of complex structures such as an umbilical can be detected by including into it a distribution of electroactive elements acting as sensors. The example consists of an ideally one-dimensional nonlinear model of an elastic umbilical containing a periodic distribution of piezoelectric elements. Local failure is accounted for via Volokh’s energy limiters approach. Finally, the solution of the model is addressed by means of the asymptotic homogenization method.

Index Terms - umbilicals; failure detection; electroactive elements; homogenization

1 INTRODUCTION

In an early review paper on flexible risers, after summarizing the problems and unknowns related to the aspects that could lead to failure with possible fatal outcomes, MacFarlane (1989) ends by stating:

“I take a wide view of flexible pipes and class them generally with umbilicals and ropes/tethers. I believe that a generic numerical model (incorporating wherever possible analytical solutions) can be built now to define the mechanical (static and dynamic), and thermal and electrical performance of these ligaments and perhaps, later, to take into account chemical degradation.

The future problem is to design complex multi-function pipes and cables from first principles and predict the performance before building the prototype.”

Since then, a great amount of efforts was devoted to the theoretical modeling of the behavior of such complex structures. For instance, a special double issue on flexible risers of the journal Marine Structures (Vol. 5, Issues 2-3, 1992) presented a collection of papers covering the academic and engineering studies carried out in the decade before that publication. Among them, Witz and Tan (1992ab) presented analytical models for the axial-torsional and flexural structural behaviors of flexible pipes, umbilicals and marine cables, with good agreement with experimental data. Seyed and Patel (1992) presented a rigorous mathematical approach to the effects of pressure and internal flow on the behavior of flexible risers, and solve analytically the cases of simple catenary, steep-S, lazy-S, steep-wave and lazy-wave risers. By adopting a general analytical approach, Owen et al. (1992) focused on the effects of global loads on the local stresses and strains and their outcomes in the failure prediction.

Another special issue on structural mechanics and service life predictions of marine cables, umbilicals and flexible pipes was published by the journal Engineering Structures (Vol. 17, Issue 4, 1995). Among them, Patel and Seyed (1995) reviewed the modeling and analysis methods for the mechanical behavior of flexible riser to that date. Witz and Tan (1995) modeled marine cables and umbilicals subjected to rotary bending and found that failure spreads uniformly along the specimens in agreement with experimental results. McIver (1995) presented a detailed mathematical modeling method for the local and global structural behavior of flexible pipe sections taking into account the effects of several phenomena such as friction, curvature deformation, wall pressure and temperature. Feld et al. (1995) extended the theory of cable mechanics to account for non-trivial strain distributions, interacting axial and bending loads, and plastic nonlinearities, to model the mechanical behavior of metallic elements of submarine cables.

Smith and MacFarlane (1996) developed a general approach to the static behavior of flexible pipes and cables by means of homotopic methods. Custódio and Vaz (2002) formulated a model for the axisymmetric behavior of umbilicals and flexible pipes subjected to various types of monotonic loads taking into account material nonlinearities, gaps and contact effects. Saevik and Bruaseth (2005) applied a theoretical-experimental approach to study the axisymmetric behavior of umbilicals with complex cross-sections and developed the corresponding finite-element formulation taking into account material nonlinearities, gaps and contact effects. Alfano et al. (2009) proposed a numerically derived constitutive model for flexible risers by using a multiscale approach in the Euler-Bernoulli beam model framework and validated it by comparison to detailed finite-element simulations. Bahtui et al. (2010) extended the previous work by including the effects of energy dissipation due to frictional slip between layers and the hysteretic response. S. Saevik (2011) presented one model to predict stresses from axisymmetric effects and two formulations to predict bending stresses in tensile armour lay-
ers of non-bonded flexible pipes in compliance with the nonlinear finite-element framework and complemented with experimental results. Ostergaard et al. (2012) studied the specific failure mode known as lateral wire buckling occurring in the tensile armor layers of flexible pipes by means of a mathematical model based on the curved beam equilibrium and allowing large deflections in compression and bending.

Despite the large number of published papers addressing the theoretical-numerical-experimental study of risers and umbilicals, when failure assessment is the subject at hand, most works are limited to modeling and prediction as mentioned above or, the less, to scheduled inspections (MacFarlane 1989; Out et al. 1995). To the best of our knowledge, little has been done to assess failure in a real-time fashion (Fang and Lyons 1992; Lyons et al. 1996; Lyons et al. 2003; Trarieux et al. 2006).

Here, by means of a simple example, it is illustrated the idea that global failure of complex structures such as an umbilical can be detected by including into it a distribution of electroactive elements acting as sensors. The example consists of an ideally one-dimensional nonlinear model of an elastic umbilical containing a periodic distribution of piezoelectric elements. Local failure is accounted for via Volokh’s energy limiters approach (Volokh 2007, 2008). Finally, the solution of the model is addressed by means of the asymptotic homogenization method (see, for instance, Bakhvalov and Panasenko, 1989). In this point, it should be emphasized that the aim of this paper is that of proposing an approach to the problem of real-time assessment of structural failure in umbilicals, and not to solve any real problem, whose complexity would lead us far from our intention of illustrating the proposed approach in a simple, straightforward manner. So, the example chosen to illustrate the proposed approach is a highly idealized one in order to be simple enough to be fully solved theoretically.

This work is structured as follows: Section 2 contains the statement of the problem and its solution via the asymptotic homogenization method; Section 3 presents an example with a particular application of the theoretical results presented in the previous section; and Section 4 contains the concluding remarks to this work.

2 Theoretical Preliminaries

2.1 Problem statement

Consider the ideal situation of an umbilical in vertical position and subjected only to the effects of gravity. Once the umbilical is deployed, it is expected to operate in a failure-free fashion over its whole lifetime. However, as the umbilical exhibits complex internal structure and functionalities, it is necessary to adopt and implement a real-time structural health monitoring policy in order to guarantee its structural integrity and correct operation. Then, the umbilical should be equipped with sensing elements to perform such a real-time monitoring. Therefore, in our ideal situation, it is supposed that the umbilical contains, along its length, a periodic distribution with period \( \alpha \), \( 0 < \alpha \ll 1 \), of electroactive elements, which will act as sensors to detect abnormal deformations and stresses that would lead to failure. In particular, only elastic deformations are allowed.

It is assumed that mechanical and electric behaviors are uncoupled in the whole umbilical except for the electroactive elements. Then, by noticing that the length-to-diameter aspect ratio is large, the umbilical is modeled as a two-phase one-dimensional periodic structure with an elastic phase (denoted with superscript \( e \)) and a electroactive phase (denoted with superscript \( a \)), and with the recurrent element \( \Omega \) characteristic length \( |\Omega|_x = \alpha \), as shown in Fig. 1. Notice that \( \{ \Omega; \emptyset \} = \Omega^e \cup \Omega^a \), where \( \Omega^A \) is the domain occupied by phase \( A \). Also notice that, in addition to the global spatial variable \( x \), a second variable \( y = \alpha^{-1} x \) is introduced in order to account for local behavior, so the recurrent element is of unit length in the local variable, that is, \( |\Omega|_y = 1 \). Moreover, with \( 1 = |\Omega|_y = |\Omega^e \cup \Omega^a|_y = |\Omega^e|_y + |\Omega^a|_y \) and the notation \( |\Omega^A|_y = c^A \) for phase concentration, it follows that \( c^e + c^a = 1 \).

With such considerations, separation of scales is well defined having variables \( x \) and \( y \) to account for the behaviors in the global and local scales, respectively.
Figure 1. Idealization of the umbilical as a one-dimensional structure containing a periodic distribution of electroactive elements.

Let $\sigma \equiv \sigma_{11}$ and $D \equiv D_3$ be the stress and the electric displacement, respectively, along the umbilical. Then, the electromechanical equilibrium is stated as

$$\sigma' + \phi = 0 \quad \text{and} \quad D' = 0$$

(1)

where $\phi$ is the gravity force and $A' \equiv A_x + \alpha^{-1}A_y$ denotes the total spatial derivative calculated using the chain rule with $A_x$ and $A_y$ the derivatives of $A$ with respect to $x$ and $y$, respectively.

Let $\varepsilon \equiv \varepsilon_{11}$ and $E \equiv E_3$ be the strain and the electric field, respectively, along the umbilical. Then, its local constitutive behavior is stated via the general nonlinear electromechanical relations

$$\sigma = \sigma(y; \varepsilon, E) \quad \text{and} \quad D = D(y; \varepsilon, E)$$

(2)

with

$$A(y) = 1^e(y)A^e + 1^a(y)A^a$$

(3)

where $1^A$ is the characteristic function of phase $A$, that is,

$$1^A(y) = \begin{cases} 1 & \text{if } y \in \Omega^A \\ 0 & \text{otherwise} \end{cases}$$

(4)

In terms of the constitutive energy functional $\psi \equiv \psi(y; \varepsilon, E)$, (2) become

$$\sigma = \frac{\partial \psi}{\partial \varepsilon} \quad \text{and} \quad D = -\frac{\partial \psi}{\partial E}$$

(5)

The mechanical displacement $u \equiv u_1$ and the electric potential $\phi$ are related to the strain $\varepsilon$ and the electric field $E$, respectively, via the relations

$$\varepsilon = u' \quad \text{and} \quad E = -\phi'$$

(6)

Continuity of fluxes and potentials across the phase interfaces is also assumed, so

$$\sigma = 0, \quad u = 0, \quad u = 0 \quad \text{and} \quad \phi = 0$$

(7)

where $A$ denotes the jump of $A$ when passing through the interfaces.

In what follows, it is handy to adopt a unified notation for the problem just-stated. Let $f \equiv (\sigma, D)$, $g \equiv (\varepsilon, E)$ and $p \equiv (u, -\phi)$ be the vectors of flux-type, gradient-type and potential-type magnitudes, and let $\phi \equiv (\phi, 0)$ be the vector of forces. Then, the problem stated above is rewritten via the equilibrium equation

$$f' + \phi = 0$$

(8)

with the local constitutive relation

$$f \equiv f(y; g) = \partial_g \psi(y; g)$$

(9)

where $\partial_g$ is the differential operator defined as

$$\partial_g A = \left( \frac{\partial A}{\partial \varepsilon}, -\frac{\partial A}{\partial E} \right),$$

(10)

and the continuity conditions

$$f = 0 \quad \text{and} \quad p = 0$$

(11)

2.2 The asymptotic homogenization method

This method addresses the solution of a differential problem with rapidly oscillating magnitudes by means of a two-scale asymptotic series expansion in terms of powers of the oscillating magnitudes.
tions’ small period $\alpha$. The unknown series coefficients are functions of the global and local spatial variables $x$ and $y$. Then, the separation of scales allows uncoupling the original problem into a sequence of problems to find the unknown coefficients of the asymptotic series of the solution. Finally, the homogenized problem, which is independent of the local variable $y$, is obtained together with the so-called effective law that relates the global dependent and independent field variables. For more details on the method see, for instance, Bakhvalov and Panasenko (1989).

The solution of problem (8)-(11) is sought as an asymptotic series in terms of powers of $\alpha$, that is,

$$ p = \pi^{(0)}(x) + \alpha^k \pi^{(k)}(x, y) $$

where $\pi^{(0)}$ and $\pi^{(k)}$, $k \geq 1$, are unknown vector functions which are 1-periodic in variable $y$, and Einstein’s summation convention over repeated indexes was adopted.

Substitution of series (12) into relation (10) yields

$$ g = \pi^{(0)}_x + \pi^{(1)}_y + \alpha^k \left( \pi^{(k)}_x + \pi^{(k+1)}_y \right) $$

Now, substitution of (13) into the (first two terms of the) Taylor series of (9) with center $(y; \pi^{(0)}_x + \pi^{(1)}_y)$ yields

$$ f(y; g) = f(y; \pi^{(0)}_x + \pi^{(1)}_y) + $$

$$ + \alpha \left( \pi^{(1)}_y + \pi^{(2)}_y \right) \partial_g f(y; \pi^{(0)}_x + \pi^{(1)}_y) + $$

$$ + O(\alpha^2) $$

where

$$ \partial_g f = \begin{pmatrix} \partial \sigma_x & \partial \sigma_x \\ \partial E_x & \partial E_x \\ \partial D_x & \partial D_x \end{pmatrix} $$

is a Jacobian matrix. Next, putting series (14) into the equilibrium equation (8) yields

$$ \alpha^{-1} \left[ f(y; \pi^{(0)}_x + \pi^{(1)}_y) \right]_y + $$

$$ + \left[ \left( \pi^{(1)}_x + \pi^{(2)}_x \right) \partial_g f(y; \pi^{(0)}_x + \pi^{(1)}_y) \right]_y + $$

$$ + \left[ f(y; \pi^{(0)}_x + \pi^{(1)}_y) \right]_x + \varphi + O(\alpha) = 0 $$

As $\alpha$ is arbitrarily small, in order to (16) be satisfied asymptotically, the coefficients of $\alpha^{-1}$ and $\alpha^0$ are equated to zero, leading to two vector differential equations, that is,

$$ f(y; \pi^{(0)}_x + \pi^{(1)}_y)_y = 0 $$

and

$$ \left[ \left( \pi^{(1)}_x + \pi^{(2)}_x \right) \partial_g f(y; \pi^{(0)}_x + \pi^{(1)}_y) \right]_y + $$

$$ + \left[ f(y; \pi^{(0)}_x + \pi^{(1)}_y) \right]_x + \varphi = 0 $$

Similarly, substitution of series (14) and (12) into the continuity conditions (11) yields four necessary conditions for (11) to be satisfied asymptotically, that is,

$$ \left[ p^{(1)} \right] = 0 \text{ and } \left[ p^{(2)} \right] = 0 $$

$$ \left[ f(y; \pi^{(0)}_x + \pi^{(1)}_y) \right] = 0 $$

and

$$ \left[ \left( \pi^{(1)}_x + \pi^{(2)}_x \right) \partial_g f(y; \pi^{(0)}_x + \pi^{(1)}_y) \right] = 0 $$

Notice that (17), (19), and (20) define a parametric family of problems with parameter $\pi^{(0)}_x$ to find $p^{(1)}$, whose uniqueness is guaranteed by imposing the condition $\langle p^{(1)} \rangle = 0$, where $\langle A \rangle = \int_0^1 A(y)dy$ is the local averaging operator. Also notice that $\langle p^{(k)} \rangle = 0$ as $p^{(k)}$ is 1-periodic in $y$. Moreover, notice that $\langle 1^A \rangle = c^A$.

Similarly, (18), (19)$_2$ and (21) constitute a parametric family of problems with parameters $\pi^{(0)}_x$ and $p^{(1)}$ to find $p^{(2)}$, whose uniqueness is guaranteed by imposing the condition $\langle p^{(2)} \rangle = 0$.

In order to complete the second order approximation $p \approx \pi^{(0)} + \alpha p^{(1)} + \alpha^2 p^{(2)}$, the first term $\pi^{(0)}$ must be provided. Then, averaging of (18) taking the 1-periodicity in $y$ of $p^{(1)}$ and $p^{(2)}$ into account yields the homogenized equilibrium equation

$$ \left( f(y; \pi^{(0)}_x + \pi^{(1)}_y) \right)_x + \langle \varphi \rangle = 0 $$
from which $\pi^{(0)}$ is obtained.

Notice that the effective global behavior follows the law $\hat{\mathbf{f}}(\mathbf{y};\pi^{(0)}+p^{(1)})$. Also notice that, averaging the series (12)-(14) yields $\hat{\mathbf{p}} = (\pi^{(0)})$, $\hat{\mathbf{g}} = (\pi^{(0)})$ and $\hat{\mathbf{f}} = (\mathbf{f}(y;\pi^{(0)}+p^{(1)}))$, respectively. Then, the effective law is written as

$$\hat{\mathbf{f}} = \hat{\mathbf{f}}(\hat{\mathbf{g}})$$

or, more specifically,

$$\hat{\mathbf{f}} = \hat{\mathbf{f}}(\hat{\mathbf{g}})$$

where \(\hat{\mathbf{A}}(\hat{\mathbf{y}};\hat{\mathbf{E}}+u^{(1)} \hat{\mathbf{E}}-\phi^{(1)} \hat{\mathbf{E}})\). For more details on the presented derivation, see Pérez-Fernández (2010).

3 A Simple Example

Here, it is assumed that the energy functional $\psi^e$ of the elastic phase of the umbilical corresponds to linear behavior for small to moderate strains, that is, $\psi^e \approx W^e$, where

$$W^e = \frac{1}{2} \left(C^{e} e^{2} - \kappa^{e} E^{2}\right)$$

with $C^{e} \equiv C_{1111}^{e}$ and $\kappa^{e} \equiv \kappa_{33}^{e}$ being the elastic and dielectric properties, respectively, of the elastic phase. In addition, it is assumed that $\psi^e$ converges to certain limiter $\Phi^e$ as the strains grow unbounded, that is, $\psi^e \xrightarrow{\varepsilon \to \infty} \Phi^e$. Such a behavior can be stated as (Volokh 2007, 2008)

$$\frac{\psi^e}{\Phi^e} = 1 - \exp \left\{ -\frac{W^e}{\Phi^e} \right\}$$

On the other hand, the electroactive phase is taken to be a linear piezoelectric material, that is,

$$\psi^a = W^a = \frac{1}{2} C^{a} e^{2} - \varphi^{a} e \phi - \frac{1}{2} \kappa^{a} E^{2}$$

where $C^{a} \equiv C_{1111}^{a}$, $\varphi^{a} \equiv \varphi_{3111}^{a}$ and $\kappa^{a} \equiv \kappa_{33}^{a}$ are the elastic, piezoelectric and dielectric properties, respectively, of the piezoelectric phase.

Now, consider that the elastic phase behaves as Araldite in the small-to-moderate strains regime while the piezoelectric elements are made of PZT-5A. Table 1 shows the relevant materials properties and concentrations employed in the numerical calculations. Linear materials properties were taken from Castilero et al. (1998).

<table>
<thead>
<tr>
<th>Property</th>
<th>$C^e$ (GPa)</th>
<th>$\rho^e$ (C/m²)</th>
<th>$\kappa^e$ ($\times 10^{-12}$)</th>
<th>$\Phi^e$ (N/m²)</th>
<th>$\varphi^e$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Araldite</td>
<td>5.46</td>
<td>0</td>
<td>7.0</td>
<td>170</td>
<td>99</td>
</tr>
<tr>
<td>PZT-5A</td>
<td>121</td>
<td>-5.4</td>
<td>830</td>
<td>-</td>
<td>1</td>
</tr>
</tbody>
</table>

$\kappa_0 = 8.85 \times 10^{-12}$ C²/Nm²

4 Concluding Remarks

By means of an example, and despite its lack of realism due to the simplification of assuming one-dimensionality and rather
simple material behavior, it was successfully illustrated the idea that it is possible to detect the global failure of a structure in a real-time fashion by including a distribution of sensing electroactive elements into it. It is emphasized that this was the aim of this paper: not to solve any real situation but proposing an approach to the problem of real-time assessment of structural failure in umbilicals, and illustrating it in a simple, straightforward manner.

ACKNOWLEDGMENT

Financial support provided by Brazilian CAPES PNPD via the project entitled “Segurança e Confiabilidade em Operações da Indústria Offshore” and the reviewers’ useful comments are both gratefully acknowledged.

REFERENCES


