Optimization under Uncertainties in Oil Reservoir using Local Surrogate Models

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Abstract—The search for optimum conditions in the production of oil and gas is one of the major challenges of petroleum engineering. The need to make fast decisions, associated with a number of limitations, such as simulations of high computational cost, require large investments in developing new computational tools. This paper presents a computational methodology for dynamic allocation of the rates in the production and injection wells. The uncertainties related to the petrophysical properties such as permeability will be considered in the rate allocation problem. The "black oil" model is used in this work, and reservoir simulations are made with a commercial simulator. The permeability field is considered as a stochastic field, featuring uncertainty as an input variable of the model. The stochastic input fields are described with the Karhunen-Loève expansion, and the stochastic responses of interest are expressed with polynomial chaos expansion. However, the use of Karhunen-Loève expansion involves a high computational cost. To overcome this problem kernel technique is used. In this work surrogate models are used, to reduce further the computational cost. Another advantage of using surrogate models is that they allow optimization studies when there is no information about the gradients of the equations or when these gradients are too expensive to obtain. The surrogate models are used together with the strategy named sequential approximation optimization (SAO). This strategy decomposes the optimization process into subproblems where each subproblem is restricted to a region of design space (trust region). The layered and nested methodology is used to perform optimization under uncertainties. Waterflooding case studies on a model reservoir are shown. The results show that the use of surrogate models is a viable alternative when the gradients of the problem are not accessible. The optimization cases of dynamic allocation of production rates shows that the methodologies presented here have achieved robust results.

Index Terms—Oil Reservoir Engineering, Waterflooding, Optimization under Uncertainty.

I. INTRODUCTION

The search for optimum conditions for the production of oil and gas is one of the major challenges of petroleum engineering. The need to make fast decisions, associated with a number of limitations, such as simulations of high computational cost, requires large investments in developing new computational tools.

Often in engineering and reservoir management, decisions need to be taken based on data obtained by numerical reservoir simulations. Several properties of the reservoir, such as porosity and permeability need to be known exactly to perform the simulation, which is not a realistic assumption in most cases. In fact, initial estimates of reservoir properties are developed using data that comes from wells, which are often separated from each other by hundreds of meters. It is important to note that the drilling of one well involves high costs, and in the initial studies few wells are drilled. There are also indirect measurements, such as seismic analysis which provides properties across the field. This paper presents computational tools for the dynamic allocation of the rates in the production and injection wells (Oliveira, 2006; Lira Jr., 2012). The uncertainties related to the petrophysical properties such as permeability field are considered. The aim of this paper is to present a computational system which provides data to assist in the management process of oil and gas reservoirs. In this system, optimization and uncertainty quantification of the reservoir production will be developed in an integrated way.

The "black oil" model is used in this work, and the simulations are done with the commercial simulator IMEX (CMG, 2006). The permeability field is considered as a stochastic field, featuring uncertainty as an input variable of the model. The stochastic input fields are described with the of Karhunen-Loève expansion (KLE) (Tatang, 1995, Huang et al, 2001; Sarma et al, 2008), and the stochastic responses of interest are expressed with polynomial chaos expansion (PCE) (Xiu and Karniadis, 2003, Eldred and Webster, 2008).
As the KLE requires high computational resources, kernel technique (KKLE) is used to minimize this drawback. The optimization process is performed using the layered and nested methodology and the SAO strategy (Giunta and Eldred, 2000; Eldred et al, 2002; Afonso, 2008; Lira Jr, 2012) in conjunction with Kriging surrogate models (Jones et al, 1998; Guinanta and Watson, 1998, Forrester et al, 2008). The SAO methodology decomposes optimization process into subproblems where each subproblem is restricted to a region of the design space (trust region). The construction of surrogate models allows a reduction in the computational cost. Another advantage of using surrogate models is that they allow to develop optimization studies when there is no gradient information or when they are too expensive to obtain. This work was developed with the use of Dakota program (Eldred et al, 2008), which is an open source system for optimization, uncertainty quantification, parameter estimation and reliability, among other capabilities for analysis of large-scale problems in engineering. The KKLE routines, together with the interface between the Imex and Dakota were developed in Matlab / Octave platforms. Waterflooding case studies in oil reservoir are shown. The results show that the methodologies presented is a viable alternative when the gradients of the problem are not accessible or a prohibitive cost is involved in their calculations.

II. DEFINITION OF OPTIMIZATION PROBLEM

The optimization problem in oil reservoirs presented in this paper is a non-linear optimization case with linear and nonlinear constraints. As already mentioned, the uncertainties of the permeability field will be considered in the optimization process. The formal statement of the problem is presented below:

Maximize \( f(x_{p,t}, u) \)

Subject to:

\[ \sum_{p \in P} x_{p,t} = 1, \ t = 1...n_t \] (2)

\[ \sum_{p \in \Omega} x_{p,t} = 1, \ t = 1...n_t \] (3)

\[ x_{p,t}^l \leq x_{p,t} \leq x_{p,t}^u \] (3)

Where:

\( f \) is the objective function;

\( x_{p,t} \) represents the control variables of the problem, \( p \) is well number, and \( t \) is the time interval;

\( u \) represents the state variables of the problem;

\( x_{p,t}^l \) and \( x_{p,t}^u \) represent the upper and lower bounds of the control variables, and \( n_t \) is the total number of time intervals of operation.

The objective function chosen in this work is the net present value (NPV) for operation of the reservoir, presented with the following simplified model (Oliveira, 2006).

\[ NPV = f(x_{p,t}, u) = \sum_{\tau=0}^{T} \frac{1}{(1+d)^\tau} F_\tau(x_{p,t}, u) \] (4)

Where \( d \) is the discount rate, \( T \) is the concession period and \( F \) is cash flow at time \( \tau \), which represents the oil revenue minus the cost of water injection and water production.

The design variables are the rates in the wells, in each time interval. The design variables are scaled by their respective maximum allowed platform rates:

\[ x_{p,t} = \frac{q_{p,t}}{Q_{b,t, max}}, \ p \in P \ ; \ x_{p,t} = \frac{q_{p,t}}{Q_{b,t, max}}, \ p \in I \] (5)

Where \( q_{p,t} \) is the rate of the well \( p \) in time interval \( t \), \( Q_{b,t, max} \) is the maximum rate of production fluids (oil and water) allowed and \( \epsilon \) is the maximum injection rate allowed on the platform, \( P \) is the set of production wells and \( I \) is the set of injection wells.

The state variables \( u \) are the parameters that can not be controlled and may have uncertainty, such as the properties of fluids, rocks, rock-fluid interaction, location of wells, in addition to the economic parameters (costs, value of the dollar). As already mentioned, uncertainties considered in this work are related to the permeability field of the reservoir. The optimization constraints are the rates limits of production and injection wells.

III. REPRESENTATION OF THE PERMEABILITY FIELD

In this work, the permeability of the rock reservoir is an uncertain parameter. However, adopting the rock properties of every cell of the numerical model that represent the reservoir as uncertain variables is incorrect because the petrophysical properties of the reservoir are spatially correlated. To overcome that, the permeability field \( y \) will be a stochastic field and represented with a spectral decomposition through the Karhunen-Loève expansion (KLE) (Tatang, 1995; Huang et al, 2001; Sarma et al, 2008). In this work linear form for KL expansion will be used.
3.1 Linear KL.

Supposing there is a set of centered realizations of a random field, given by \( y_k, k = 1, ..., N_r \), where \( N_r \) is the number of realizations and each vector has \( N_e \) elements, corresponding in the case of reservoir simulations to the value of the property of interest in the cell of the reservoir. The covariance matrix of the input field can be computed with

\[
C = \frac{1}{N_r} \sum_{j=1}^{N_e} y_j y_j^T
\]  

(6)

where it is expected that the number of realizations is large enough to ensure convergence of the matrix. Through the KLE, new realizations of the field \( y \) can be computed in terms of the eigenvalues and eigenvectors of the covariance matrix, as in:

\[
y = E \Lambda^{1/2} \xi
\]  

(7)

In which \( \xi \) is a vector of random standard normal variables and \( E, \Lambda \) are respectively, the eigenvector and the diagonal eigenvalue matrix of the covariance matrix given in Eq. (6). According to Scholkopf (1996) to obtain the KLE the following eigenvector/eigenvector equation needs to be solved.

\[
\hat{\lambda} v = C v
\]  

(8)

The solution of Eq. (8) could be very costly as the dimension of \( C \) is \( N_e \times N_r \), where \( N_r \) is the number of cells in the model. This problem can be overcome applying a kernel technique (Sarma, 2008) in which the eigenvector and eigenvalues presented in Eq. (7) are obtained from the solution of a reduced order \( (N_e \times N_e) \) eigenvalue/eigenvector problem. The procedure is briefly described below. Pre and post multiplying Eq. (6) by \( v \), leads to:

\[
C v = \frac{1}{N_r} \sum_{j=1}^{N_e} (v_j, v_j) y_j
\]  

(9)

and considering Scholkopf (1996) assumptions in which the eigenvectors can be written as linear combination of provided realizations:

\[
v = \sum_{j=1}^{N_e} \alpha_j y_j
\]  

(10)

Combining Equations 8, 9 and 10 leads the following form:

\[
2 \sum_{i=1}^{N_e} \alpha_i (y_i, y_i) = \frac{1}{N_r} \sum_{j=1}^{N_e} \alpha_j \left( \sum_{i=1}^{N_e} \alpha_i (y_i, y_i) (y_j, y_j) \right)
\]  

(11)

Defining \( K \) as the kernel matrix (dimension \( N_r \times N_r \)), from which each element is obtained as

\[
K_{ij} = (y_i, y_j)
\]  

(12)

Where \( y_i, y_j \) are realizations belonging to set \( y = \{ y_1, y_2, ..., y_{N_r} \} \). Through the above definition, Eq. (11) can be written in a compact form such as

\[
N_r \hat{\lambda} K \alpha = K \alpha
\]  

(13)

Which leads to:

\[
N_r \lambda K \alpha = K \alpha
\]  

(14)

The above equation is known as the kernel eigenvector/eigenvalue equation (Scholkopf et al, 1996). Solving such equation the solution of Eq. (8) will also be obtained using the relation of Eq. (10). Moreover Eq. (14) involves a problem of dimension \( N_e \times N_e \), while the original one Eq. (8) has dimension \( N_r \times N_r \), as in general \( N_r \ll N_e \) the solution of Eq. (8) is obtained in a much more efficient way.

IV. POLYNOMIAL CHAOS EXPANSION

The polynomial chaos expansion allows an expansion of second-order random process in terms of Hermitian polynomials. The polynomials are function of Gaussian random variables ( Eldred, 2008). Polynomial chaos is an expression in terms of orthogonal polynomials on stochastic variables, consisting of an infinite summation of the polynomial degree, and infinite summation of the number of random variables of orthogonal polynomials. A second-order random variable \( S(\theta) \) can be represented as follows, where \( \theta \) is an independent event (Xiu et al, 2003):

\[
S(\theta) = a_0 \phi_0 + \sum_{i=1}^{N} a_i \phi_i (\xi_1 (\theta)) + \sum_{i=1}^{N} a_i \phi_i (\xi_2 (\theta), \xi_1 (\theta)) +... 
\]  

(15)

In above expression, \( \phi_n (\xi_1, ..., \xi_N) \) are the Hermitian polynomials of order \( n \), and \( \xi_i, i = 1, ..., N \), is an independent Gaussian random variables, with zero mean and unit variance. The general expression for the Hermitian polynomials \( \phi_n \) is given by (Xiu et al., 2003):
\[ \phi_i(\xi_1, \ldots, \xi_n) = \exp \left( -\frac{1}{2} \sum_{i=1}^{n} \xi_i^2 \right) \exp \left( \sum_{i=1}^{n} \frac{1}{2} \frac{\partial^2}{\partial \xi_i \partial \xi_j} \right) \]  

Equation 15 may be rewritten in a more conventional form such as an infinite summation of terms shown in Eq. (17).

\[ S(\xi) = \sum_{i=1}^{\infty} \alpha_i \Phi_i(\xi) \]  

In this equation, \( \alpha_i \) are the coefficients to be determined, and the polynomials \( \Psi_i(\xi) \) are known, being products of one-dimensional orthogonal polynomials. Equations 15 and 17 are exact equalities, in probabilistic terms, but are not practical for use, because they depend on an infinite number of random variables and polynomials of infinite degree. In practice, truncations are applied on both directions. It uses a finite number of variables and the degree of the polynomial is also limited, and generally limited to a relatively low level. Eq. (17) is rewritten as:

\[ S(\xi) \approx \sum_{i=1}^{\hat{p}} \sum_{j=1}^{p} \alpha_{ij} \Psi_{ij}(\xi). \]  

It is now clear that the summation is only an approximation to the unknown function, but it is shown (Xiu et al, 2003) that for functions that meet certain requirements, convergence is very fast.

In this work, the calculation of the expansion coefficients is done using the linear regression method. In this method the coefficients are calculated solving a linear least squares problem of the form:

\[ \Psi \alpha = S \]  

Where \( \Psi \) is a vector with the coefficients to be determined, the vector \( S \) contains the unknown function values calculated for a set of predetermined points (collocation points). The matrix \( \Psi \) contains the polynomial evaluated at these points. The total number of function evaluations with this technique is given by:

\[ N = 2^{n+p}, \]  

Where \( n \) is the number of uncertain variables and \( p \) is the degree of the polynomial used.

V. SURROGATE MODELS

Surrogate models are built to provide smooth functions with sufficient accuracy for fast evaluations for both uncertainty quantification and in the optimization process. Kriging (Forrester et al, 2008; Simpson, 2001) methodology is considered in this work. The central idea of this scheme is to assume that errors in the model are not independent but rather exhibit spatial correlation related to the distance between corresponding points modeled by a Gaussian process around each sample point.

The main advantages of this scheme are easily to accommodate irregularly distributed sample data, and the ability to model multimodal functions with many peaks and valleys. Moreover, kriging models provide exact interpolation at the sample points.

Kriging is a data fitting based model, the first step is to generate the sampling points. These points in the design space are determined by DOE (Giunta, 2002) approach. In this work Latin Centroidal Voronoi Tessellations (LCVT) is used (Du et al, 1999). Next, the kriging fitting scheme is used to develop the predictor and error expressions in order to evaluate the functions at untried design points.

In the present work we consider the ordinary kriging approach which models the true function as

\[ f(x) = \beta + Z(x) \]  

where \( \beta \) is an unknown constant and \( Z(x) \) is the standard Gaussian random distribution with zero mean and variance one. The covariance matrix of \( Z(x) \) is given by:

\[ \text{Cov}[Z(x_i), Z(x_j)] = \sigma^2 R(\|x_i - x_j\|) \]  

where \( x_i \) and \( x_j \) are the sample points and the correlation function \( R \) is given by:

\[ R(x_i, x_j) = \exp \left( -\sum_{k=1}^{n_d} \theta_k |x_i^{(k)} - x_j^{(k)}|^2 \right) \]  

where \( n_d \) is the total number of variables, and \( \theta_k \)'s are the unknown correlation parameters used to fit the model. Predicted estimates, \( \hat{f}(x) \) of the response \( f(x) \) at untried values are given by

\[ \hat{f}(x) = E(\{ f(x) \} | f(x^{(i)}), \ldots, f(x^{(n_d)}) ) \]  

which means from statistics the expected value of \( f(x) \), given the information in \( f(x^{(i)}), \ldots, f(x^{(n_d)}). \) A measure of the amount of the error between the kriging model \( \hat{f}(x) \) and the true model \( f(x) \) in all design space location can be found introducing the concept of mean squared error (MSE) where

\[ \text{MSE} = E(\hat{f}(x) - f(x))^2 \]
The predicted estimates \( \hat{f}(x) \) of the response \( f(x) \) at any point of the design domain can be obtained by minimizing the MSE, leading to

\[
\hat{f}(x) = \beta + r^T(x)R^{-1}(f_x - \hat{\beta}f)
\]  
(26)

where \( f_x = [f_1, ..., f_m] \) are the true values of the function at the \( m \) points, \( f_x = [1, ..., 1] \) is a vector of ones, and:

\[
r(x) = [R(x, x_1), ..., R(x, x_m)] \text{ and } \hat{\beta} = (r^T R^{-1} r)^{-1} r^T R^{-1} f_x
\]  
(27)

In Equation 27 both \( r \) and \( R \) depend on the unknown parameter \( \theta_k \) as indicated in Eq. (23). This is found using maximum likelihood estimation (Forrester et al., 2008). In kriging models some assessment strategies are required to check a priori if a generated model is adequate. Such measure will also provide some guidelines for selecting the best model when different models are constructed. In this work, two strategies are considered. They are RMSE (Root Mean Square Error) and PRESS (Predicted Error Sum of Squares) (Forrester et al., 2008; Afonso et al., 2008).

**VI. OPTIMIZATION STRATEGY**

To tackle the problem formulated in section 2 the SAO methodology (Forrester et al., 2008; Keane and Nair, 2005; Alexandrov et al., 1997) is employed. In this procedure, surrogate smooth functions are used in place of the computationally expensive and/or nonsmooth functions in a sequence of optimization steps, that are confined to a small region of the parameter space. To update the design space of each optimization subproblem a trust region based scheme (Eldred, 2004; Alexandrov et al., 1997) is used.

Mathematically, each subproblem \( k \) is defined as:

Maximize \( \hat{j}^k(x) \)

Subject to:

\[ g_i^k(x) \leq 0, \quad i = 1, ..., m \]

\[ x_l^k \leq x \leq x_u^k \]

Where:

\[ x_l^k = x_l^k - \Delta^k \]

\[ x_u^k = x_u^k + \Delta^k \]

In above equations, \( \hat{j}^k(x) \) and \( g^k(x) \) are respectively the approximated (surrogate) objective and constraints functions, \( x_l^k \) is the center point of the trust region, \( \Delta^k \) is the width of the trust region and \( x_l^k, x_u^k \) are respectively the lower and upper bounds of the design variables for SAO iteration \( k \).

6.1 SAO Algorithm

Each subproblem described above defines a SAO iteration.

To update the trust region size \( \Delta^k \) for each optimization subproblem we considered the approach described in Alexandrov et al (1997) which takes into consideration the accuracy of surrogate functions against the true functions. The main steps involved in the computations are:

1. Compute the expensive and/or nonsmooth objective function and constraints at the central point in the subregion.
2. Construct surrogate model in the subregion.
3. Optimize within the subregion using the surrogate objective function and constraints.
4. Compute the true objective function and constraints at the optimum identified in Step 3.
5. Check for convergence.
6. Move/shrink/expand the subregion according to the accuracy of the approximated model compared to the true function and constraint values.
7. Impose constraint consistency.
8. Check for overall optimization convergence. If it is achieved stop the SAO procedure; otherwise return to Step 3.

6.2 Trust Region Updating Scheme

To update the trust region size for each optimization subproblem we considered the approach described in reference (Giunta et al., 2000). A parameter, \( \rho_k \), controls the trust region size. This parameter measures the accuracy of the surrogate function at \( x_c^k \) and is calculated as

\[
\rho^k = \min(\rho_{f}^k, \rho_{g}^k), \quad \text{for } k = 0, 1, 2, k_{\text{max}}
\]  
(29)

Where:

\[
\rho_{f}^k = \frac{f(x_c^k) - f(x_c^k)}{\hat{f}(x_c^k) - \hat{f}(x_c^k)} \quad \text{and} \quad \rho_{g}^k = \frac{g(x_c^k) - g(x_c^k)}{\hat{g}(x_c^k) - \hat{g}(x_c^k)}
\]  
(30)

The next trust region size is updated as follows:

\[
\Delta^{k+1} = \begin{cases} 
0.5\Delta^k, & \text{if } \rho^k \leq 0, \\
0.5\Delta^k, & \text{if } 0 < \rho^k \leq 0.25, \\
\Delta^k, & \text{if } 0.25 < \rho^k < 0.75 \text{ or } \rho^k > 1.25, \\
2\Delta^k, & \text{if } 0.75 \leq \rho^k \leq 1.25
\end{cases}
\]  
(31)

The next iterate \( x_c^{k+1} \), is given by:

\[
x_c^{k+1} = \begin{cases} 
x_c^k, & \text{if } \rho^k > 0 \\
x_c^k, & \text{if } \rho^k \leq 0
\end{cases}
\]  
(32)
In which \( f \) and \( g \) are the true high fidelity objective and constraint functions.

**VII. OPTIMIZATION UNDER UNCERTAINTY**

Many optimization problems should be solved considering the uncertainties due to lack of knowledge concerning the properties of real physical problem. The challenge is to include these uncertainties in the optimization procedures with efficiency and reliability.

In developing procedures for optimization under uncertainties (OUU), optimization techniques must be combined with techniques for uncertainty quantification (Eldred et al, 2002). In this paper, the techniques for local optimization with surrogates models are used (SAO - Sequential Approximate Optimization), together with uncertainty quantification (UQ) techniques. The inputs of the uncertainty quantification process are the permeability fields. These inputs are parameterized with the Karhunen-Loeve expansion (Tatang, 1995, Huang et al, 2001; Sarma et al, 2008). The outputs will be represented with the polynomial chaos expansion (Xiu and Karniadis, 2003, Eldred and Webster, 2008).

The OUU formulation presented in this work is known as layered and nested (Eldred et al, 2002). This formulation is based on the use of surrogate models (data fitting), statistics are calculated from the uncertainty quantification step. The step of UQ is performed for each combination of control variables provided by the optimizer. The optimization studies developed in this work were performed using the computational package Dakota (Eldred, 2008).

Figure 1 presents a diagram of the methodology layered and nested, according to the scheme: \( S_u \) are the responses of the objective function in the surrogate model, \( u \) is the stochastic state variable (for example, the permeability field), \( e r_u \) is the response of high-fidelity model (simulator), \( S_u \) is the objective, for example, the expected NPV, \( d \) are the control variables, which in this work are the liquid rates in production wells.

With this methodology it is possible to perform optimization studies in oil reservoirs, where the gradients of the problem are not available or require high computational resources to be obtained, in addition, uncertainties in petrophysical properties are considered.

The following is a summary of the optimization under uncertainty process applied in this work:

1. A number of well rate samples are obtained in the trust region by the LCVT method (Du et al, 1999).
2. For each sample an uncertainty quantification process is developed, and the expected NPV is obtained.
3. A surrogate model is generated in the trust region.
4. An optimization process using sequential quadratic programming is developed in the surrogate model.
5. The response in the high-fidelity model is calculated using optimal control variables found in step 4.
6. The convergence of the process is verified.
7. The trust region is modified, according to a comparison between values of the high-fidelity model and the surrogate model.
8. Return to step 1.

**VIII. WATERFLOODING IN OIL RESERVOIR**

The case study presented here is a waterflooding problem in oil reservoir with one injection and two production wells, as illustrated in Fig. 2. The reservoir has an area of 510 x 510 m², with a thickness of 4 m and is modeled with a mesh 51x51x1. 1000 possible realizations for the permeability field were used. The realizations were generated considering the following covariance function (Ghanem, 1998):

\[
\text{cov} = \sigma^2 e^{\frac{(x_i - x_j)^2}{b_1} - \frac{(y_i - y_j)^2}{b_2}}
\]  

(33)

In the above equation \((x_i, y_i), (x_j, y_j)\) are the coordinates of two generic points on the grid, \(b_1\) and \(b_2\) are correlation lengths and \(\sigma^2\) is the standard deviation. The following values are attributed for the studies: \(b_1 = 6, b_2 = 9\) and \(\sigma^2 = 1\).
The production time for the reservoir is 24 years. The two producers wells are part of a platform system operating with a liquid flow of 6 m³/day. The objective function of optimization problem is the NPV (Eq. (4)), and the design variables are the liquid rate at producers that have limits of 25% and 75%. Two control cycles of production time are used, the first and second range were 8 years and 16 years respectively, this implies two design variables problem. Initially a deterministic optimization study was conducted, where one of the possible realizations was chosen (Fig. 3).

![Fig. 3. Realization chosen for the study of deterministic optimization.](image)

The deterministic optimization was performed with SAO strategy, where kriging surrogate models were used in the trust regions. The models were constructed with LCVT sampling. Table 1 presents the results of deterministic optimization. The Table shows that the NPV is maximum when the well 1 operates with a 26.4% in the first interval of time (eight years) and 70.7% in the second interval (16 years remaining). Fig. 4 shows the cumulative production of oil and water curves for the initial and optimized cases.

![Fig. 4. Cumulative production of oil and water curves for the initial and optimized cases.](image)

**Table 1. Results of deterministic optimization**

<table>
<thead>
<tr>
<th></th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>NPV $\times 10^5$ (US$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Values</td>
<td>75.0</td>
<td>25.0</td>
<td>2.385</td>
</tr>
<tr>
<td>Optimum Values</td>
<td>26.4</td>
<td>70.7</td>
<td>2.510</td>
</tr>
</tbody>
</table>

The results of deterministic optimization show an increase in NPV of 5.5% if compared with initial rates. This improvement is due to increased oil production and a reduced production of water, plus the delay of the production of water as can be seen in Fig. 4 and 5. With respect to computational cost, 72 Imex simulations were performed in parallel with 6 processors. The SAO strategy was performed with 12 iterations to reach the optimal solution.

![Fig. 5. Watercut for the initial and optimized cases.](image)

Figure 5 shows the watercut for the initial and optimized cases.

An optimization study considering the uncertainties in the permeability field was also done. A thousand possible realizations are considered using the linear KKLE method with 10 principal components. Usually a number of principal components which produce normalized cumulative sum of the magnitudes of the eigenvalues around 0.70 is a reasonable choice. In this case the objective function is the expected value of NPV. The optimization under uncertainty was performed with the layered and nested methodology, where surrogate models were generated by kriging and the generation of samples by the technique LCVT. Second order polynomial chaos was used, whose expansion coefficients were calculated by linear regression. Table 2 presents the optimization results.

**Table 2. Initial and optimum values of optimization under uncertainty.**

<table>
<thead>
<tr>
<th></th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>E (NPV) $\times 10^5$ (US$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Values</td>
<td>75.0</td>
<td>25.0</td>
<td>2.746</td>
</tr>
<tr>
<td>Optimum Values</td>
<td>75.0</td>
<td>40.5</td>
<td>2.802</td>
</tr>
</tbody>
</table>
The optimum rates in well 1 was 75% in first control cycle and 40.5% in the second. It is important to note that the result of optimization under uncertainty takes into account the 1000 possible realizations for this type of optimization, despite an increase in the computational cost, more consistent results are obtained when compared with deterministic cases. The computational cost was 4572 inmix simulations executed in parallel with 6 processors. The SAO strategy performed 6 iterations to reach the optimal solution.

In order to verify the result of optimization under uncertainty, the Monte Carlo method was used to calculate the optimal expected NPV with optimum rates, in other words, the expected NPV is calculated for a specified number of permeability field realizations. Table 3 shows the expected values (E) and standard deviations (S) obtained with the proposed method and the Monte Carlo (MC) with 1000 and 2000 realizations generated by KL expansion. The same table also shows the results for an uncertainty quantification study using polynomial chaos expansion (PCE), which can be verified the accuracy of polynomial expansion (132 simulations) compared to the Monte Carlo method (1000 and 2000 simulations). These results are obtained for the optimal rates in the process of optimization under uncertainty.

Table 3. Expected value and standard deviation of NPV for uncertainty quantification studies.

<table>
<thead>
<tr>
<th></th>
<th>PCE</th>
<th>MC – 1000</th>
<th>MC – 2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (NPV) * 10^5  (US$)</td>
<td>2.802</td>
<td>2.796</td>
<td>2.793</td>
</tr>
<tr>
<td>S (NPV) * 10^5  (US$)</td>
<td>0.201</td>
<td>0.196</td>
<td>0.202</td>
</tr>
</tbody>
</table>

The expected NPV and the standard deviation obtained show the accuracy of the polynomial chaos expansion when compared with the Monte Carlo method (Table 3). It is important to emphasize that in the PCE 132 simulations performed in the high-fidelity model were necessary, whereas the Monte Carlo method required respectively 1000 and 2000 simulations.

A study to verify the robustness of the optimization under uncertainty solution was performed (Van Essem et al., 2006). Four realizations of 1000 possible were randomly chosen. For each realization a deterministic optimization study was run. Then, the optimal rates obtained in the process of optimization under uncertainty were assigned to each of the four realizations and then the NPV was calculated. In the final step the optimal deterministic rates obtained in each of the four realizations were allocated to the other three realizations in the study. For example, the optimal rates obtained in the realization 1 were allocated in the reservoir considering the realization 2, 3 and 4. This process was repeated for all realizations, and the NPV was calculated for each case. Tables 4 and 5 present all the results of the robustness verification process. Table 4 shows the results obtained for the deterministic optimization process for each chosen realization, as well as the results obtained in the optimization under uncertainty study. In Table 5, the letter R represents a realization. In bold, the results obtained in the deterministic optimization process. Figure 6 shows the results of Table 5 in a graph form, in order to clarify the results.

Table 4. Flows rates and NPV results for the deterministic optimization and OUU process.

<table>
<thead>
<tr>
<th></th>
<th>Realization 1 (C1 Case)</th>
<th>Realization 2 (C2 Case)</th>
<th>Realization 3 (C3 Case)</th>
<th>Realization 4 (C4 Case)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rates (%)</td>
<td>[0.26, 0.70]</td>
<td>[0.25, 0.75]</td>
<td>[0.67, 0.59]</td>
<td>[0.72, 0.31]</td>
</tr>
<tr>
<td>NPV * 10^5 (US$)</td>
<td>2.510</td>
<td>2.964</td>
<td>3.059</td>
<td>2.902</td>
</tr>
</tbody>
</table>

Table 5. Results for robustness of the optimization under uncertainty process.

<table>
<thead>
<tr>
<th></th>
<th>R 1</th>
<th>R 2</th>
<th>R 3</th>
<th>R 4</th>
<th>Mean Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV OUU Case * 10^5 (US$)</td>
<td>2.460</td>
<td>2.850</td>
<td>2.990</td>
<td>2.877</td>
<td>2.794</td>
</tr>
<tr>
<td>NPV C1 Case * 10^5 (US$)</td>
<td>2.510</td>
<td>2.506</td>
<td>3.058</td>
<td>2.664</td>
<td>2.685</td>
</tr>
<tr>
<td>NPV C2 Case * 10^5 (US$)</td>
<td>2.385</td>
<td>2.964</td>
<td>2.802</td>
<td>2.893</td>
<td>2.761</td>
</tr>
<tr>
<td>NPV C3 Case * 10^5 (US$)</td>
<td>2.476</td>
<td>2.594</td>
<td>3.059</td>
<td>2.728</td>
<td>2.714</td>
</tr>
<tr>
<td>NPV C4 Case * 10^5 (US$)</td>
<td>2.418</td>
<td>2.935</td>
<td>2.888</td>
<td>2.902</td>
<td>2.786</td>
</tr>
</tbody>
</table>

Fig. 6. Graph of NPV results shown in Table 5.

The results presented in Tables 3 to 5, and also in Fig. 6
represent the robustness of the result obtained in the optimization under uncertainty process. As expected, the NPV values related to deterministic optimization are the best cases for each realization. However, in the real reservoir problems true realization of the permeability field is not known, invalidating the deterministic optimization. The NPV values obtained with the rates coming from optimization under uncertainty (OUU) are good, especially when compared with results obtained from the rates resulting from the deterministic optimization process performed in each realization.

The rates obtained in the process of optimization under uncertainty, when placed in each of the four selected realizations, producing a mean NPV of 2794 x 10^2 US$. This value is higher than the mean NPV values obtained where C1, C2, C3 and C4 using the controls rates of other realizations (Table 5). The optimal rate obtained in the optimization process under uncertainty in each case produce a satisfactory estimate of the NPV. The OUU method used prevents the worst cases of control to be chosen. This would happen, for example, if deterministic optimization of the rates were calculated on the realization 2 and the real reservoir realization was realization 1, as shown in Fig. 6. In this situation, the resulting NPV from realization 1 would be lower when compared to the NPV produced by OUU process. This same idea can be applied to the realization 2, 3 and 4 (Fig. 6).

CONCLUSION

The optimization of the production rates in oil reservoirs under uncertainty was shown here to be a highly complex problem. The challenges in this field are diverse, involving mathematical, physical and computational aspects. The main conclusions of the work are presented below:

1. The use of surrogate models is computationally feasible alternative for solving optimization problems in oil reservoir where the gradients of the problem are not available or will require high computational resources to be obtained. It is important to note that feasibility of the optimization under uncertainty problems depends on the use of parallel computation.

2. The use of the polynomial chaos expansion proved to be efficient compared with the Monte Carlo technique.

3. The optimization under uncertainty procedure presented robust results, that is, the control rates obtained in the process had certain immunity to the lack of knowledge of the real permeability field.

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REFERENCES


